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Naturally light sterile neutrinos

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A simple model to accommodate light sterile neutrinos naturally with large mixing with the usual neutrinos has been proposed. The standard model gauge group is extended to include an $SU(2)_S$ gauge symmetry. Heavy triplet Higgs scalars give small masses to the left-handed neutrinos, while a heavy doublet Higgs scalar mixes with the sterile neutrinos of the same order of magnitude. We then present a simple form of the neutrino mass matrix, which may originate from this scenario and can explain the solar neutrino deficit, the atmospheric neutrino deficit, the LSND data, and the hot dark matter. The lepton number is violated through decays of the heavy triplet Higgs scalars, which generate the lepton asymmetry of the universe, which in turn generates a baryon asymmetry of the universe. [S0556-2821(99)50203-8]

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Recently the Super Kamiokande [1] announced positive evidence of neutrino oscillations. They attribute the ν_μ deficit in the atmospheric neutrino to a ν_μ oscillating into a ν_{atm}, where ν_{atm} could be a ν_τ or a sterile neutrinos ν_S (which is a singlet under the standard model) with $\Delta m^2_{atmos} = m^2_{\nu_2} - m^2_{\nu_3} \sim (0.5\text{–}6) \times 10^{-3}\,\text{eV}^2$, where ν_2 and ν_3 are physical mass eigenstates and ν_μ and ν_{atm} are admixtures of these states with almost maximal mixing. There are also indications of neutrino oscillations in the neutrinos coming from the Sun. The solar neutrino deficit can be explained if one considers $\nu_e \to \nu_{sol}$ oscillations (where ν_{sol} could be ν_μ or ν_τ or ν_S) with the mass squared difference [2] $\Delta m^2_{solar} = m^2_{\nu_x} - m^2_{\nu_1} \sim (0.3\text{–}1.2) \times 10^{-5}\,\text{eV}^2$, where $x=2$, 3, or 4, and it is assumed that ν_e and ν_{sol} are admixtures of the mass eigenstates ν_1 and ν_x. This mass squared difference is for resonant oscillation [3]. If one assumes a vacuum oscillation solution of the solar neutrino deficit, then this number will be several orders of magnitude smaller. If we assume a three generation scenario, ν_{atm} is then identified with ν_τ, so that ν_μ and ν_τ are mostly ν_2 and ν_3, and ν_{sol} could be a ν_μ or a ν_τ. Consider $\nu_{sol} \equiv \nu_\tau$; i.e., ν_τ contains all ν_1, ν_2, and ν_3, although ν_e is dominantly ν_1, which implies, $m^2_{\nu_2} - m^2_{\nu_1} = (m^2_{\nu_2} - m^2_{\nu_3}) + (m^2_{\nu_3} - m^2_{\nu_1}) = \Delta m^2_{atmos} + \Delta m^2_{solar} \sim 10^{-2}\,\text{eV}^2$. Then we cannot explain the Liquid Scintillation Neutrino Detector (LSND) result [4], which announced a positive evidence of $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations with the mass squared difference (alternate explanation is also not possible [5]) $\Delta m^2_{\text{LSND}} = m^2_{\nu_2} - m^2_{\nu_1} \sim (0.2\text{–}2)\,\text{eV}^2$. This conclusion is true even if we consider $\nu_{sol} \equiv \nu_\mu$. Although recent preliminary results from KARMEN [6] disfavor the LSND allowed region, it is too premature to say conclusively that KARMEN rules out LSND [7]. Particularly, since the sensitivity of KARMEN is too low. Only miniBOONE can cover the entire region of LSND and tell us conclusively if there is $\nu_e \to \nu_\mu$ oscillation in this mass range. In this article, we shall point out that theoretically LSND result is not disfavored, which is important for motivating future experiments.

As a solution to this problem, one can say that either LSND result is wrong or there has to be some other explanation for the solar neutrino deficit. But a more popular solution is to extend the standard model to incorporate a sterile neutrino and explain all these experiments [8]. Since data from the CERN e^+e^- collider LEP [9] ruled out any possibility of a fourth $SU(2)_L$ doublet left-handed neutrino, this fourth neutrino has to be a sterile neutrino, which does not interact through any of the standard model gauge bosons. Incorporating such light sterile neutrino with large mixing with the other light neutrinos in extensions of the standard model is nontrivial [8]. Recently, there is one attempt to extend the radiative neutrino mass generation model by Zee [10] to incorporate a sterile neutrino [11]. However, that model cannot explain the baryon asymmetry of the universe.

In this article, we propose a new scenario with an $SU(2)$ symmetry, which can provide a naturally light sterile neutrino with large mixing with the other left-handed neutrinos. We then present a simple neutrino mass matrix with the sterile neutrinos which can have its origin from this scenario and can now explain the LSND data [4], the solar neutrino problem [2], the atmospheric neutrino anomaly [1], and the dark matter problem [12]. The lepton number violation at a very high scale generates a lepton asymmetry of the universe, which then gets converted to the baryon asymmetry of the universe during the electroweak phase transition.

We work in an extension of the standard model which includes a couple of heavy triplet Higgs scalars [14,13], whose couplings violate lepton number explicitly at a very high scale, which in turn gives small neutrino masses naturally. Decays of these triplet Higgs scalars generates a lepton asymmetry of the universe [13]. We extend this minimal scenario to include a $SU(2)_S$ gauge group so as to extend the standard model gauge group to

$$\mathcal{G}_{ext} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(2)_S .$$

The $SU(2)_S$ symmetry breaks down along with the lepton number at very high energy (M), and the out-of-equilibrium

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condition for generating baryon asymmetry of the universe determines this scale M [16]. Since all representations of the $SU(2)$ groups are pseudoreal and anomaly free, there is no additional constraints coming from cancellation of anomaly. This makes this mechanism easy to implement in different scenarios.

The fermion and the scalar content of the standard model, which transform under $SU(3)_c\times SU(2)_L\times U(1)_Y$ as

$$q_{iL}\equiv(3,2,1/6),\quad u_{iR}\equiv(3,1,2/3),\quad d_{iR}\equiv(3,1,-1/3),$$

$$l_{iL}\equiv(1,2,-1/2),\quad e_{iR}\equiv(1,1,-1),\quad \phi\equiv(1,2,1/2)$$

are all singlets under the group $SU(2)_S$. $i=1,2,3$ is the generation index. The two heavy triplet Higgs scalars $\xi_a \equiv(1,3,1)$, $a=1,2$, required to give masses to the left-handed neutrinos are also singlets under $SU(2)_S$. In this mechanism, we add a $SU(2)_S$ doublet neutral left-handed fermion S_L and two scalars η and χ, which transform under $\mathcal{G}_{ext}$ as

$$S_L\equiv(1,1,0,2),\quad \eta\equiv(1,2,1/2,2),\quad \chi\equiv(1,1,0,2).$$

There are two scales in the theory, the $SU(2)_S$ and the lepton number violating scale M and the electroweak symmetry breaking scale m_W. At a high energy M, χ acquires a vacuum expectation value (VEV) and breaks $SU(2)_S$. Lepton number is broken explicitly at this scale through the couplings of the scalar triplets. All the new scalars are considered to be very heavy,

$$M_\eta\sim M_\chi\sim M_{\xi_a}\sim M.$$

The fields η and ξ_a do not acquire any VEV. However, once the standard model Higgs doublet ϕ acquires a VEV, these fields ξ_a and η acquires a very tiny VEV, which in turn gives very small masses and large mixing to the neutrinos.

Consider the most general potential of all the scalars in the model (ξ_a,η,χ,ϕ). There will be quadratic and quartic couplings of the form

$$M_H^2(H^\dagger H),\lambda_H(H^\dagger H)(H^\dagger H)\quad\text{and}\quad \lambda'_{12}(H_1^\dagger H_1)(H_2^\dagger H_2),$$

where H, H_1, and H_2 correspond to any of the scalar fields. In addition, there will be two coupled terms:

$$\begin{aligned}V=&\mu_a(\xi_a^0\phi^0\phi^0+\sqrt{2}\xi_a^-\phi^+\phi^0+\xi_a^{--}\phi^+\phi^+)\\&+m[\phi^0(\eta_+^0\chi_--\eta_-^0\chi_+)-\phi^+(\eta_+^-\chi_--\eta_-^-\chi_+)]\\&+m[\phi^0(\eta_+^0\chi_-^*-\eta_-^0\chi_+^*)-\phi^+(\eta_+^-\chi_-^*-\eta_-^-\chi_+^*)]+\text{H.c.},\end{aligned}\tag{1}$$

where η_+^- represents the component of η with electric charge -1 and $T_3=+1/2$ of $SU(2)_S$; χ_+ represents the component of χ with $T_3=+1/2$ of $SU(2)_S$; and χ_+^* is the component of $\chi^\dagger$ with $T_3=+1/2$ of $SU(2)_S$.

For consistency [13], we require μ_a to be less than but of the order of masses of ξ_a, and we choose

$$\mu\sim 0.1\quad M.$$

When the field χ acquires a VEV, a mixing of the fields ϕ and η of amount $m\langle\chi\rangle$ will be induced. Since $M_\eta\sim M$ and $m_\phi\sim m_W$, to protect the electroweak scale, we then require

$$m\sim m_W.$$

This choice of m has a more natural origin in supersymmetric version of the present model. The scalar trilinear term may originate from a scalar superpotential

$$W=m_\phi\phi^\dagger\phi+\lambda\phi\eta\chi$$

which gives us $m=\lambda m_\phi\sim m_W$, which is the natural scale of m.

This fixes all the mass parameters in this scenario. We can now proceed to minimize the potential. In Ref. [13] it was shown that the triplet Higgs scalars get a very small VEV consistent with the minimization of the potential. In the present scenario, both the Higgs triplet ξ_a and the new doublet Higgs scalar η get a tiny VEV on minimization, without any fine tuning of parameters. We assume that $T_3=+1/2$ component of χ acquires a VEV. But that can induce VEVs to both the neutral $SU(2)_S$ components η_+^0 and η_-^0. They are given by

$$\langle\xi_a\rangle\simeq-\frac{\mu\langle\phi\rangle^2}{M_{\xi_a}^2},$$

$$\langle\eta_-^0\rangle\simeq-\frac{m\langle\phi\rangle\langle\chi_+\rangle}{M_\eta^2}\quad\text{and}\quad\langle\eta_+^0\rangle\simeq-\frac{m\langle\phi\rangle\langle\chi_-^*\rangle}{M_\eta^2}.\tag{2}$$

Since, $\mu\sim M_{\xi_a}\sim M_\eta\sim\langle\chi_+\rangle\sim M$ and $m\sim m_\phi\sim\langle\phi\rangle\sim m_w$, we get,

$$\langle\xi_a\rangle\sim\langle\eta_-^0\rangle\sim\langle\eta_+^0\rangle\sim O\left(\frac{m_W^2}{M}\right).\tag{3}$$

The VEVs of ξ_a now give small masses to the left-handed neutrinos and the VEV of $\eta_\pm^0$ allows mixing of the $SU(2)_L$ singlet neutrinos S_L with the usual left-handed neutrinos, both of which are now of the same order of magnitude naturally.

The Yukawa couplings of the leptons are given by,

$$\mathcal{L}=f_{\alpha i}^e\overline{l_{iL}}e_{\alpha R}\phi+f_{aij}l_{iL}l_{jL}\xi_a+h_{ix}\epsilon_{xy}l_{iL}S_{Lx}\eta_y+\text{H.c.},\tag{4}$$

where $x,y=1,2$ are the $SU(2)_S$ indices. The first term contributes to the charged lepton masses, while the second and third terms contribute to the neutrino mass and mixing matrices. In the basis, $[\nu_{iL}S_{Lx}]$, we can now write down the mass matrix as

$$\mathcal{M}_\nu=\begin{pmatrix}\Sigma_a f_{aij}\langle\xi_a\rangle & h_{ix}\epsilon_{xy}\langle\eta_y^0\rangle\\ h_{ix}^T\epsilon_{xy}\langle\eta_y^0\rangle & 0\end{pmatrix}.\tag{5}$$

There are no Majorana mass terms for the sterile neutrinos. We shall now discuss how to generate baryon asymmetry of

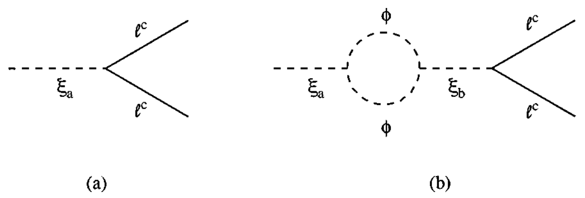

FIG. 1. The decay of $\xi_a \to l^c l^c$ at tree level (a) and in one-loop order (b). CP-violation comes from an interference of these diagrams.

the universe [15,16] in this scenario and what constraint it gives on the new mass scale M and then come back to the neutrino mass matrix.

Lepton number is violated when the scalars ξ_a decay:

$$\xi_a \to \begin{cases} l_i^c l_j^c & (L=-2), \\ \phi\phi & (L=0). \end{cases} \tag{6}$$

All other couplings conserve lepton number. By assigning a lepton number -1 to S_{Lx}, we can ensure conservation of lepton number in the decays of the doublet scalar field η without loss of generality.

We choose the mass matrix of ξ_a to be real and diagonal $\begin{pmatrix} M_{\xi_1} & 0 \\ 0 & M_{\xi_2} \end{pmatrix}$; but once the one loop self-energy type contributions are included, imaginary phases from μ_a and f_{aij} makes it complex. The absorptive part of the one loop self-energy type diagram will introduce observable CP violation in the mass matrix [17], which would produce unequal amounts of leptons and anti-leptons in the decays of the ξ_a^{++} and ξ_a^{--}, respectively. This will create a charge asymmetry, which will be compensated by equal and opposite amount of charge asymmetry in the production of ϕ^+ and ϕ^- in the decays of ξ_a^{++} and ξ_a^{--}, so that the universe remains charge neutral.

The interference of the tree level and the one loop diagram of Fig. 1 will generate a lepton asymmetry in the decays of ξ_a, given by,

$$\delta_a \simeq \frac{Im(\mu_1 \mu_2^* \Sigma_{k,l} f_{1kl} f_{2kl}^*)}{8\pi^2 (M_{\xi_1}^2 - M_{\xi_2}^2)} \left(\frac{M_{\xi_a}}{\Gamma_{\xi_a}} \right), \tag{7}$$

where, the decay width of these scalars ξ_a is given by

$$\Gamma_{\xi_a} = \frac{1}{8\pi} \left(\frac{|\mu_1|^2 + |\mu_2|^2}{M_{\xi_a}} + \sum_{i,j} |f_{aij}|^2 M_a \right). \tag{8}$$

These decays should be slower [16] than the expansion rate of the universe (H), otherwise the lepton asymmetry δ_a in decays of ξ_a will be suppressed by an amount K $(\ln K)^{0.6}$, where $K = \Gamma_{\xi_a}/H$; $H = \sqrt{1.7 g_*}(T^2/M_{Pl})$ at $T = M_{\xi_a}$; M_{Pl} is the Planck scale; and g_* is the total number of relativistic degrees of freedom.

We consider $M_{\xi_2} < M_{\xi_1}$, so that when ξ_2 decays, ξ_1 has already decayed away and only the asymmetry δ_2 generated in decays of ξ_2 will contribute to the final lepton asymmetry of the universe. The lepton asymmetry thus generated will be the same as the $(B-L)$ asymmetry of the universe, which will then get converted to a baryon asymmetry during the electroweak phase transition [18]. The final baryon asymmetry of the universe is given by

$$\frac{n_B}{s} \sim \frac{\delta_2}{3 g_* K (\ln K)^{0.6}}. \tag{9}$$

To obtain the desired amount of baryon asymmetry of the universe, one possibility could be as follows [13]: We consider, $M_2 = 10^{13}$ GeV, and $\mu_2 = 2 \times 10^{12}$ GeV, which gives $m_{\nu_\tau} = 1.2 f_{233}$ eV, assuming that the M_1 contribution is negligible. If $M_1 = 3 \times 10^{13}$ GeV, $\mu_1 = 10^{13}$ GeV, and $f_{1kl} \sim 0.1$, the decay of ψ_2 generates a lepton asymmetry δ_2 of about 8×10^{-4} if the CP phase is maximal. Using $M_{Pl} \sim 10^{19}$ GeV and $g_* \sim 10^2$, we find $K \sim 2.4 \times 10^3$, so that $n_B/s \sim 10^{-10}$.

Thus, with the heavy mass scale to be of the order of $M \sim 10^{13-14}$ GeV, it is possible to get the desired amount of baryon asymmetry of the universe and VEVs of ξ and η to be of the order of a few eV. Then with proper value of the Yukawa couplings f_{aij} and h_{ix}, we can get a neutrino mass matrix [Eq. (5)] which can explain all the neutrino experiments. All the elements of the mass matrix could be nonzero and are about a few eV or less, except for the Majorana mass term of the sterile neutrinos. One can then have several possible scenarios [8] which can explain all the neutrino experiments. Consider for example [19], one sterile neutrino with mass of about $(2-3) \times 10^{-3}$ eV, which mixes with the ν_e, while the other sterile neutrino is several orders of magnitude lighter. This will satisfy the constraints on the sterile neutrinos from nucleosynthesis. A better proposition to satisfy the nucleosynthesis bound would be to consider the vacuum oscillation solution of the solar neutrino problem, in which case $h_{ix} \epsilon_{xy} \langle \mathring{\eta}_y \rangle \sim 10^{-4}$ eV. We may then choose the scalar potential to get $\langle \chi_-^* \rangle / \langle \chi_+ \rangle \sim 10^{-3}$, so that the second sterile neutrino gets mass $\sim 10^{-7}$ eV. The nucleosynthesis bound may also be evaded, if a large lepton asymmetry $[\sim O(1)]$ is generated after the electroweak phase transition.

For purpose of completeness, we now present an explicit model of neutrino mass, which can explain all the experiments and can originate in the present scenario. We assume that the Yukawa couplings f_{aij} are such that the mass matrix of the $SU(2)_L$ doublet neutrinos have some texture zeros and only one of the sterile neutrinos mixes with ν_e given by the VEV of η and the coupling h_{ix}. The other sterile neutrino is much lighter, which is ensured by the choice of the potential and $\langle \chi_-^* \rangle / \langle \chi_+ \rangle \sim 10^{-3}$. The mixing angle may also be smaller and may not play any role in the present experiments, so we do not include it in the mass matrix. The mass matrix in the basis $[\nu_e \ \ \nu_\mu \ \ \nu_\tau \ \ \nu_s]$ may take one of the simple forms considered in Ref. [20], given by only four nonzero parameters,

$$M_\nu = \begin{pmatrix} 0 & 0 & m_1 & m_4 \\ 0 & m_3 & m_2 & 0 \\ m_1 & m_2 & 0 & 0 \\ m_4 & 0 & 0 & 0 \end{pmatrix}, \tag{10}$$

where we assume, $m_4 < m_{1,3} \ll m_2$. Consider a representative set of values for these parameters [20],

$$m_1 \sim .05 \text{ eV}; \; m_2 \sim 1.5 \text{ eV}; \; m_3 \sim 0.001 \text{ eV};$$

$$m_4 \sim 0.0001 \text{ eV}.$$

The different masses and mixing has been calculated in this case and the relevant mass squared differences and the corresponding mixing angles are given as follows [20].

The four mass eigenvalues are, $m_{\nu_{1,4}} \sim 0.0001\,\text{eV}$ and $m_{\nu_{2,3}} \sim 1.5$ eV. $m_{\nu_{2,3}}$ can contribute to the hot dark matter of the universe. The different mass squared differences and the relevant mixing angles (a) $m^2_{\nu_1} - m^2_{\nu_2} \sim 2\,\text{eV}^2$ with $\sin^2 2\theta_{e\mu} = 0.001$ explains the LSND result; (b) $m^2_{\nu_2} - m^2_{\nu_3} \sim 0.003\,\text{eV}^2$ with $\sin^2 2\theta_{\mu\tau} = 1$ explains the atmospheric neutrino problem and (c) $m^2_{\nu_1} - m^2_{\nu_4} \sim 2 \times 10^{-10}\,\text{eV}^2$ with $\sin^2 2\theta_{\mu\tau} = 1$ gives the vacuum oscillation solution to the solar neutrino problem. It is to be noted that in models without a sterile neutrino, it is not possible to get such simple texture of the mass matrix with few parameters which can only explain the solar and the atmospheric neutrino problems.

To summarize, we propose a simple scenario to accommodate naturally light sterile neutrinos in extensions of the standard model with an $SU(2)_S$ symmetry. There are two mass scales in the model, the electroweak scale and the scale of lepton number and $SU(2)_S$ violation, which is fixed by the conditions for lepton asymmetry of the universe. The heavy triplet and a doublet acquires very tiny VEV, which gives masses and mixing of the left-handed neutrinos and the sterile neutrinos. We then present a representative mass matrix, which can originate in this scenario, which explains the solar neutrino deficit, the atmospheric neutrino anomaly, the LSND result and the dark matter problem. The decays of the triplets generates a lepton asymmetry of the universe, which gets converted to a baryon asymmetry of the universe before the electroweak phase transition.

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